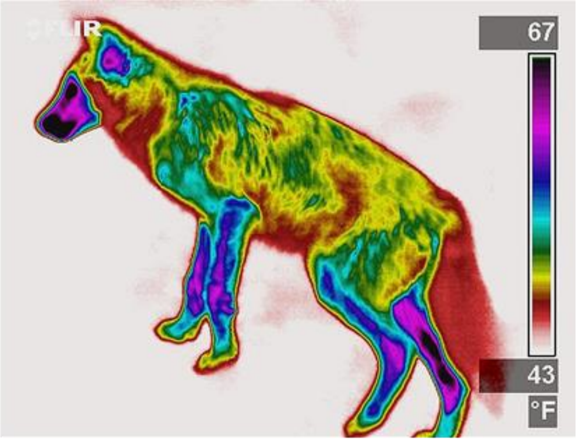
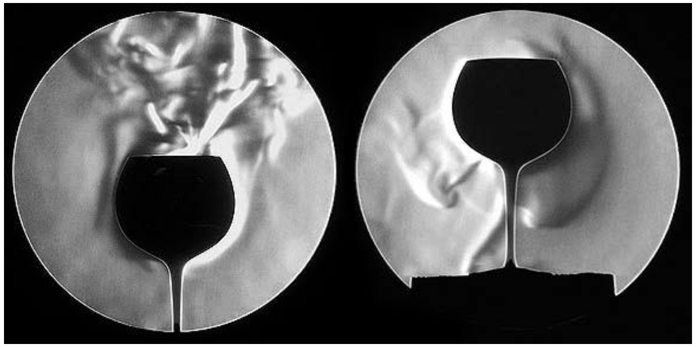
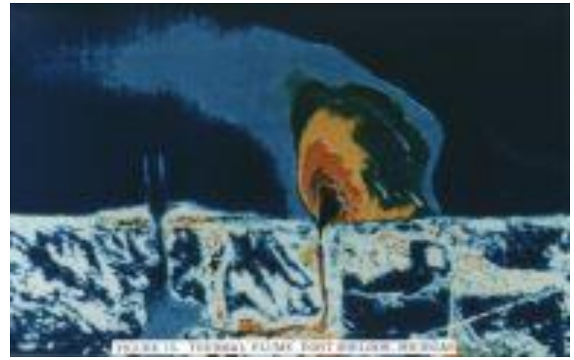
Heat Transport



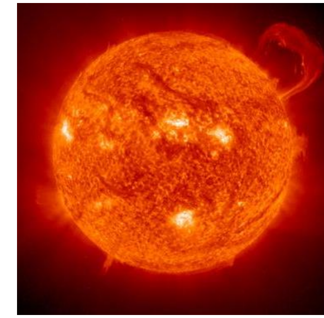
Temperature of a wolf pup



Goblet filled with coffee and ice water



Thermal plume, Lake Michigan

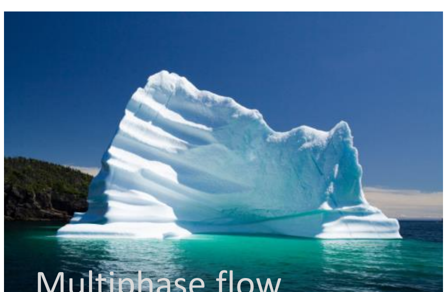


Solar radiation

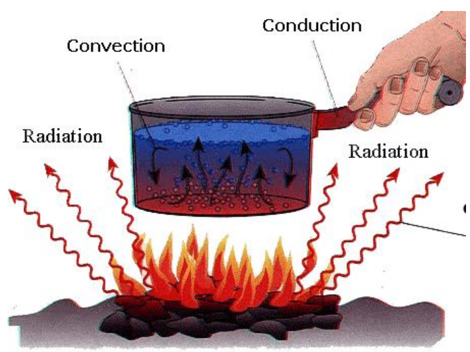
Radiating steel bar

Temperature distribution

Schlieren image of natural convection.
Note laminar-turbulent transition



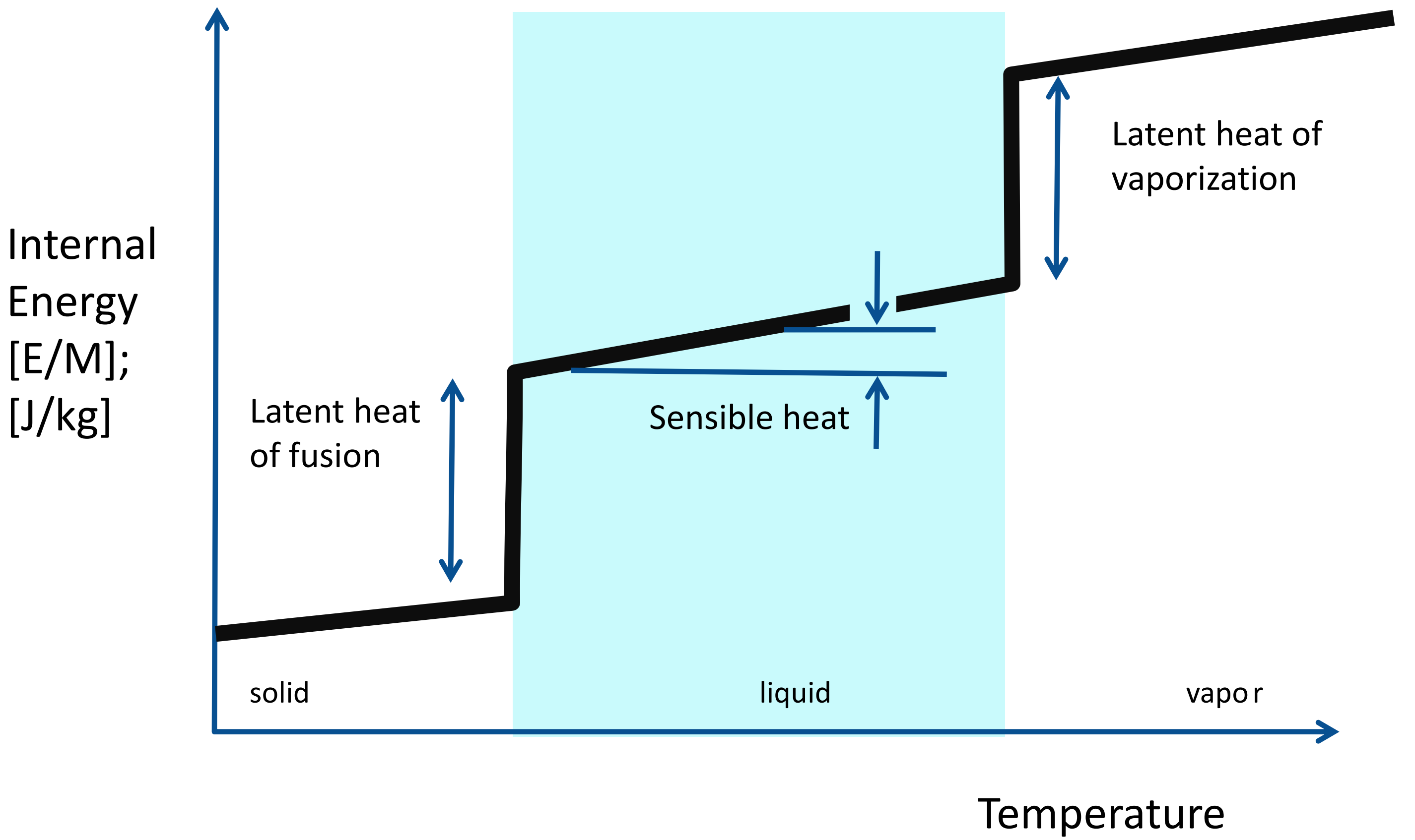
Multiphase flow



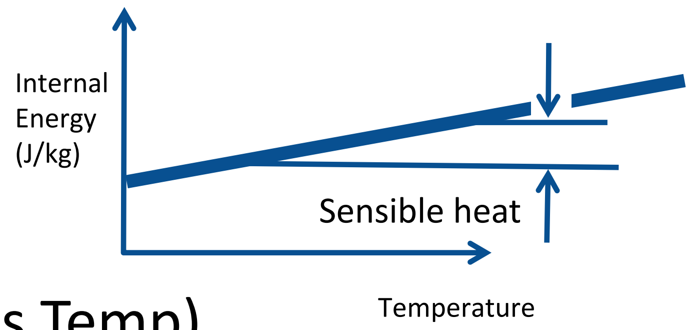
Concepts

- Storage
- Source
- Flux

Heat Storage



Heat Storage as *Sensible* Heat



c_p=specific heat capacity = Energy/(Mass Temp)

Change of sensible heat

$$\Delta E = c_p \Delta T \qquad \left[\frac{E}{M\theta}\frac{\theta}{\ }\right]$$

θ: temperature

On a per volume basis

$C_p\rho$=Energy/(Vol Temp)

$$c_p \rho \Delta T \qquad \left[\frac{E}{M\theta}\frac{M}{L^3}\frac{\theta}{\ }\right]$$

Specific Heat Capacity

Table of specific heat capacities at 25 °C (298 K) unless otherwise noted
Notable minima and maxima are shown in maroon

Substance	Phase	(mass) specific heat capacity c_p or c_m J·g^{-1}·K^{-1}	Constant pressure molar heat capacity $C_{p,m}$ J·mol^{-1}·K^{-1}	Constant volume molar heat capacity $C_{v,m}$ J·mol^{-1}·K^{-1}	Volumetric heat capacity C_v J·cm^{-3}·K^{-1}
Air (Sea level, dry, 0 °C (273.15 K))	gas	1.0035	29.07	20.7643	0.001297
Air (typical room conditions[A])	gas	1.012	29.19	20.85	0.00121
Aluminium	solid	0.897	24.2		2.422
Ammonia	liquid	4.700	**80.08**		3.263
Animal tissue (incl. human)[21]	mixed	3.5			3.7*
Antimony	solid	0.207	25.2		1.386
Argon	gas	0.5203	20.7862	12.4717	
Arsenic	solid	0.328	24.6		1.878
Beryllium	solid	1.82	16.4		3.367
Bismuth[22]	solid	0.123	25.7		1.20
Cadmium	solid	0.231	26.02		
Carbon dioxide CO_2[17]	gas	0.839*	36.94	28.46	
Chromium	solid	0.449	23.35		
Copper	solid	0.385	24.47		3.45
Tungsten[22]	solid	0.134	24.8		2.58
Uranium	solid	**0.116**	27.7		2.216
Water at 100 °C (steam)	gas	**2.080**	**37.47**	**28.03**	
Water at 25 °C	liquid	**4.1813**	**75.327**	**74.53**	**4.1796**
Water at 100 °C	liquid	**4.1813**	**75.327**	**74.53**	**4.2160**
Water at −10 °C (ice)[22]	solid	**2.11**	**38.09**		**1.938**
Zinc[22]	solid	0.387	25.2		2.76
Substance	Phase	C_p J/(g·K)	$C_{p,m}$ J/(mol·K)	$C_{v,m}$ J/(mol·K)	Volumetric heat capacity J/(cm^3·K)

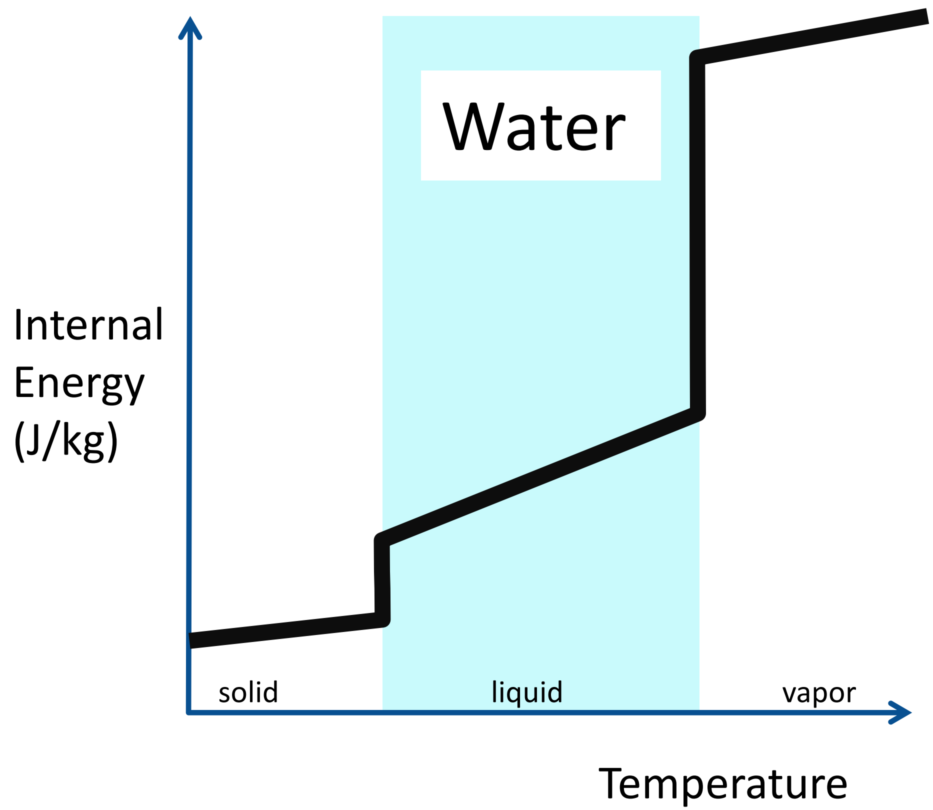
[A] Assuming an altitude of 194 metres above mean sea level (the world–wide median altitude of human habitation), an indoor temperature of :
(40.85% relative humidity), and 760 mm–Hg sea level–corrected barometric pressure (molar water vapor content = 1.16%).

	kJ/(kg K)
Water:	4.18
Ice:	2.11
Air:	1.00
CO2:	0.84
Gold:	0.13
Hydrogen:	14 (max)

Span factor of 100
Water in upper 90%

Heat Storage as *Latent* Heat

Substance	Latent Heat Fusion kJ/kg	Melting Point °C	Latent Heat Vaporization kJ/kg	Boiling Point °C
Alcohol, ethyl	108	−114	855	78.3
Ammonia	339	−75	1369	−33.34
Carbon dioxide	184	−78	574	−57
Helium			21	−268.93
Hydrogen(2)	58	−259	455	−253
Lead[8]	23.0	327.5	871	1750
Nitrogen	25.7	−210	200	−196
Oxygen	13.9	−219	213	−183
R134a		−101	215.9	−26.6
Toluene	72.1	−93	351	110.6
Turpentine			293	
Water	334	0	2260	100



Melting water:	334	kJ/kg
Heating, 0-100°C:	420	kJ/kg
Vaporing water:	2260	kJ/kg

Heat Sources

Source: Einar Helland Berger (Wikimedia Commons)
Flame from combustion (burning) of a fuel.

Heat generated internally $\left[\frac{E}{L^3 T}\right]$

Friction: fluids, solids, energy dissipation

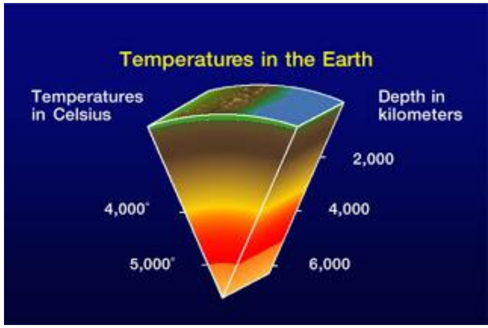
Chemical reaction: Exo, Endothermic

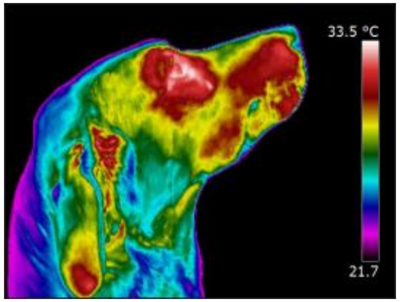
Radioactive: Fission, fusion

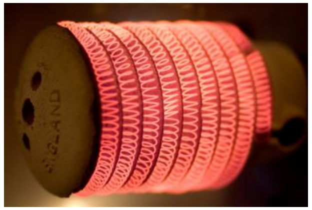
Biological: Metabolism

Dielectric: Microwaves, oven

Joule: Electrical resistance, toaster

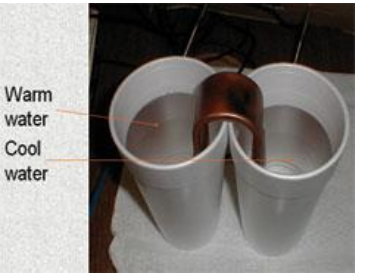






Types of heat transfer

Conduction: Static media, Diffusion



Convection: Fluid moving, advection

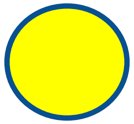
Forced;

Free or Natural

Radiation: Emit and adsorb EM, no media

Thermal flux $\frac{E}{L^2T} = \frac{Power}{L^2}; \quad \frac{J}{m^2 s} = \frac{W}{m^2}$

Radiation



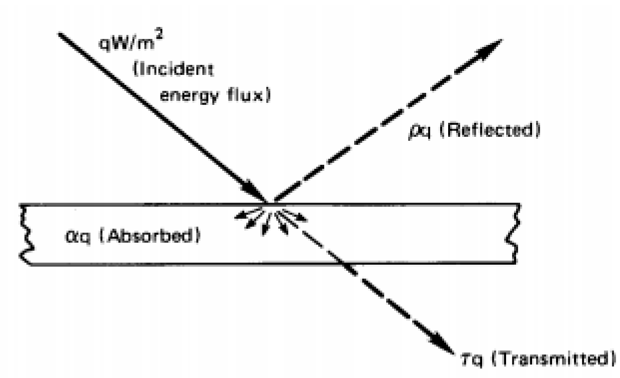


Table 1.2 Forms of the electromagnetic wave spectrum

Characterization	*Wavelength, λ*	
Cosmic rays	< 0.3 pm	
Gamma rays	0.3–100 pm	
X rays	0.01–30 nm	
Ultraviolet light	3–400 nm	***Thermal Radiation 0.1–1000 μm***
Visible light	0.4–0.7 μm	
Near infrared radiation	0.7–30 μm	
Far infrared radiation	30–1000 μm	
Millimeter waves	1–10 mm	
Microwaves	10–300 mm	
Shortwave radio & TV	300 mm–100 m	
Longwave radio	100 m–30 km	

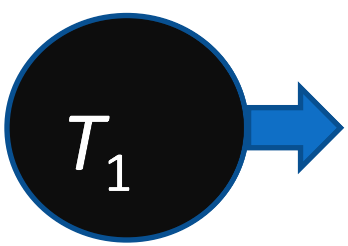
Heat transfer flux by radiation

"Black" body absorbs all incoming radiation and is a perfect emitter.

$$q_{hR} = \sigma\varepsilon(T_1^4 - T_2^4)$$

σ = Stefan-Boltzman constant
5.67x10^{-8} W/(m^2 K^4)

ε = emissivity (0-1); ~1: black; ~0: shiny



$$e_\lambda = \sigma T_1^4$$ Stefan's Law

http://www.thermoworks.com/emissivity_table.html

Thermal Conduction

$$q_{hx} = -K_h \frac{dT}{dx}$$

Fourier's Law, energy flux

$$q_{hx} = -K_h \nabla T$$

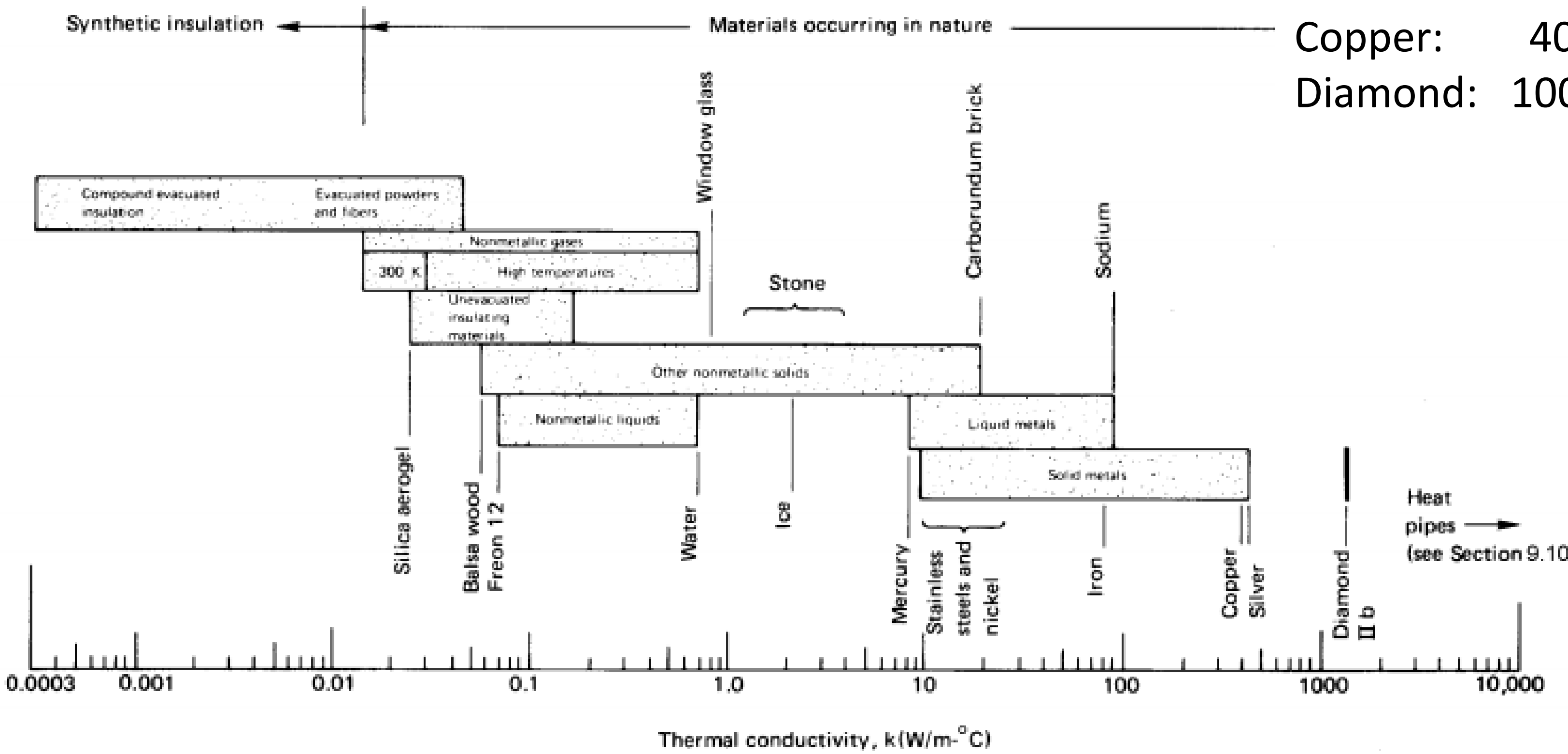
$$\left[\frac{E}{L^2 T}\right] = \left[\frac{E}{TL\theta}\frac{\theta}{L}\right]$$

$$K_h = \left[\frac{E}{TL\theta}\right] = \left[\frac{P}{L\theta}\right] = \frac{J}{ms\,^oC} = \frac{W}{m\,^oC}$$

Thermal conductivity

Thermal conductivity

	W/(m°C)
Vacuum:	0.001
Air:	0.02
Water:	0.6
Ice:	2
Steel:	10
Copper:	400
Diamond:	1000



http://web.mit.edu/lienhard/www/ahttv202.pdf

Convection

Energy flux: $\mathscr{A} = qTc_p\rho$ $\quad \frac{L_f^3}{L_c^2 T}\frac{\theta}{}\frac{E}{\theta M}\frac{M_f}{L_f^3}$

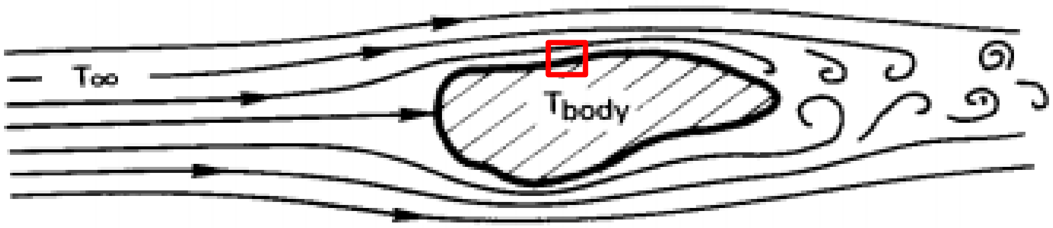
Forced: Fluid flow driven by processes other than thermal, pump etc.
Free or Natural: Fluid flow by thermally induced density differences

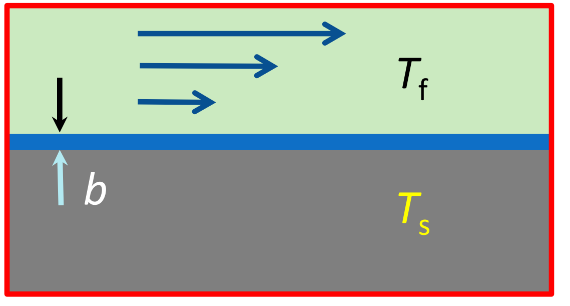
Convection

Energy flux: $\mathscr{A} = qTc_p\rho$ $\quad \frac{L_f^3}{L_c^2 T}\frac{\theta}{} \frac{E}{\theta M}\frac{M_f}{L_f^3} = \frac{}{L_c^2 T} - \frac{E}{} -$

Forced: Fluid flow driven by processes other than thermal, pump etc.
Free or Natural: Fluid flow by thermally induced density differences

Convective heat transfer —fluid←→solid





$$q_h = \frac{K_h}{b}(T_s - T_f) = \bar{h}(T_s - T_f)$$

$$\frac{E}{L^2T} = \frac{E}{L^2T\theta}\theta; \qquad \bar{h} : \left[\frac{E}{L^2T\theta} = \frac{P}{L^2\theta}; \ \frac{W}{m^2K}\right]$$

Convective Heat Transfer Coefficient

Table 1.1 Some illustrative values of convective heat transfer coefficients

Situation	$\overline{h}$, W/m^{2}K
Natural convection in gases	
• 0.3 m vertical wall in air, ΔT = 30°C	4.33
Natural convection in liquids	
• 40 mm O.D. horizontal pipe in water, ΔT = 30°C	570
• 0.25 mm diameter wire in methanol, ΔT = 50°C	4,000
Forced convection of gases	
• Air at 30 m/s over a 1 m flat plate, ΔT = 70°C	80
Forced convection of liquids	
• Water at 2 m/s over a 60 mm plate, ΔT = 15°C	590
• Aniline-alcohol mixture at 3 m/s in a 25 mm I.D. tube, ΔT = 80°C	2,600
• Liquid sodium at 5 m/s in a 13 mm I.D. tube at 370°C	75,000
Boiling water	
• During film boiling at 1 atm	300
• In a tea kettle	4,000
• At a peak pool-boiling heat flux, 1 atm	40,000
• At a peak flow-boiling heat flux, 1 atm	100,000
• At approximate maximum convective-boiling heat flux, under optimal conditions	10^6
Condensation	
• In a typical horizontal cold-water-tube steam condenser	15,000
• Same, but condensing benzene	1,700
• Dropwise condensation of water at 1 atm	160,000

Heat transfer coefficient

- *h* is not a constant, but *h* = *h*(*ΔT*).
- Three types of convection.
- Natural convection. Fluid moves due to buoyancy.

$$h \propto \Delta T^{x} \qquad \tfrac{1}{4} < x < \tfrac{1}{3}$$

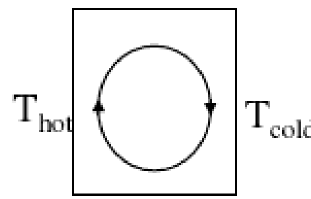
- Forced convection: flow is induced by external means.

$$h = const$$

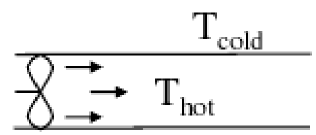
- Boiling convection: body is hot enough to boil liquid.

$$h \propto \Delta T^{2}$$

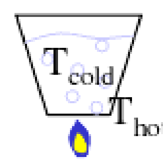
Typical values of *h*:



4 - 4,000 W/m^2-K



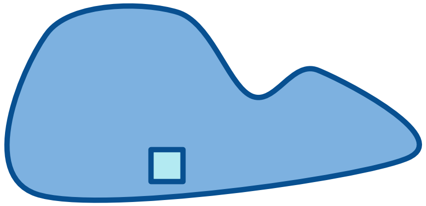
80 - 75,000



300 - 900,000

Governing Equation

Conservation of Heat Energy



$$c = \frac{E}{L_c^{\ 3}}$$

$$\nabla \cdot \Gamma + \frac{\partial c}{\partial t} = \mathcal{S}$$

$$\Gamma = \mathcal{D} + \mathcal{A}$$

Storage $c = c_p \rho n T$

$$\frac{E}{M\theta}\frac{M}{L_f^3}\frac{L_f^3}{L_c^3}\frac{\theta}{}$$

$$\frac{\partial c}{\partial t} = \frac{\partial c_p \rho n T}{\partial t}$$

Advective Flux $\mathcal{A} = c_p \rho T q$

$$\left[\frac{E}{M\theta}\frac{M}{L_f^3}\frac{\theta}{}\frac{L_f^3}{L_c^2 T}\right] = \left[\frac{E}{}\frac{}{L_c^2 T}\right]$$

Diffusive Flux (Fick's Law) $\mathcal{D} = -K_h \nabla T$

Dispersive Flux $\mathcal{D}_h = -D_h \nabla T$

Source $\mathcal{S} = \mathcal{S}_h$

$$\nabla \cdot (-D\nabla T) + \nabla \cdot \mathbf{v}T + \frac{\partial T}{\partial t} = S$$

$$D = \frac{K_h + D_{hyd}}{c_p \rho n}$$

$$S = \frac{\mathcal{S}_h}{c_p \rho n}$$

Governing $$\nabla \cdot (-(K_h + D_h)\nabla T) + \nabla \cdot \mathbf{q} c_p \rho T + \frac{\partial c_p \rho n T}{\partial t} = \mathcal{S}_h$$

Simulation of Heat Transport

$$\nabla \cdot (-D\nabla T) + \nabla \cdot \mathbf{v}T + \frac{\partial T}{\partial t} = S$$

$T = N_1$ Dirichlet, Specify T

$\mathbf{n} \cdot q_h = 0$ Neumann, Insulating, no flux

$\mathbf{n} \cdot q_h = N_2$ Specified flux

$\mathbf{n} \cdot q_h = h(T_{ext} - T)$ Convective cooling

$\mathbf{n} \cdot q_h = \varepsilon\sigma(T_{ext}^4 - T^4)$ Surface to external radiation

$\mathbf{n} \cdot q_h = \frac{K_h}{b}(T_{ext} - T)$ Cauchy-type boundary. Thin, insulating layer

Parameters

$$\nabla \cdot (-D \nabla T) + \nabla \cdot \mathbf{v} T + \frac{\partial T}{\partial t} = S$$

c_p heat capacity
ρ density
K_h thermal conductivity
q fluid flux
n porosity
D_{hyd} hydraulic dispersion

$$D_{thermal} = \frac{K_h}{c_p \rho n} = \text{thermal diffusivity} = \frac{\cancel{E}}{\cancel{L_c} T \cancel{\theta}} \frac{\cancel{M_f} \cancel{\theta}}{\cancel{E}} \frac{\cancel{L_f^3}}{\cancel{M_f}} \frac{\cancel{L_c^3}^{\,2}}{\cancel{L_f^3}}$$

$$D_{h,disp} = \frac{D_{hyd}}{c_p \rho n} = \text{thermal dispersion}$$

$$S = \frac{\mathcal{S}_h}{c_p \rho n} = \left[\frac{E}{T L_c^3} \frac{M \theta}{E} \frac{L_f^3}{M} \frac{L_c^3}{L_f^3} \right] = \left[\frac{\theta}{T} \right]$$

Coupled effects

- Chemical reaction rate
- Phase change
- Material properties
- Thermal expansion
- Solid-Fluid in porous media
- Biological growth

Temperature and Chemical Rxn

- Reaction Rate ~~Constant~~ Function
- Arrhenius eq

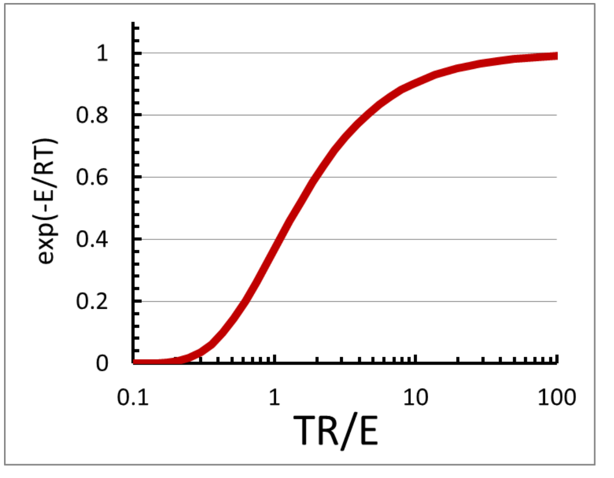
$$k = Ae^{-\frac{E_A}{RT}}$$

E_A: activation energy for reaction

R: gas constant

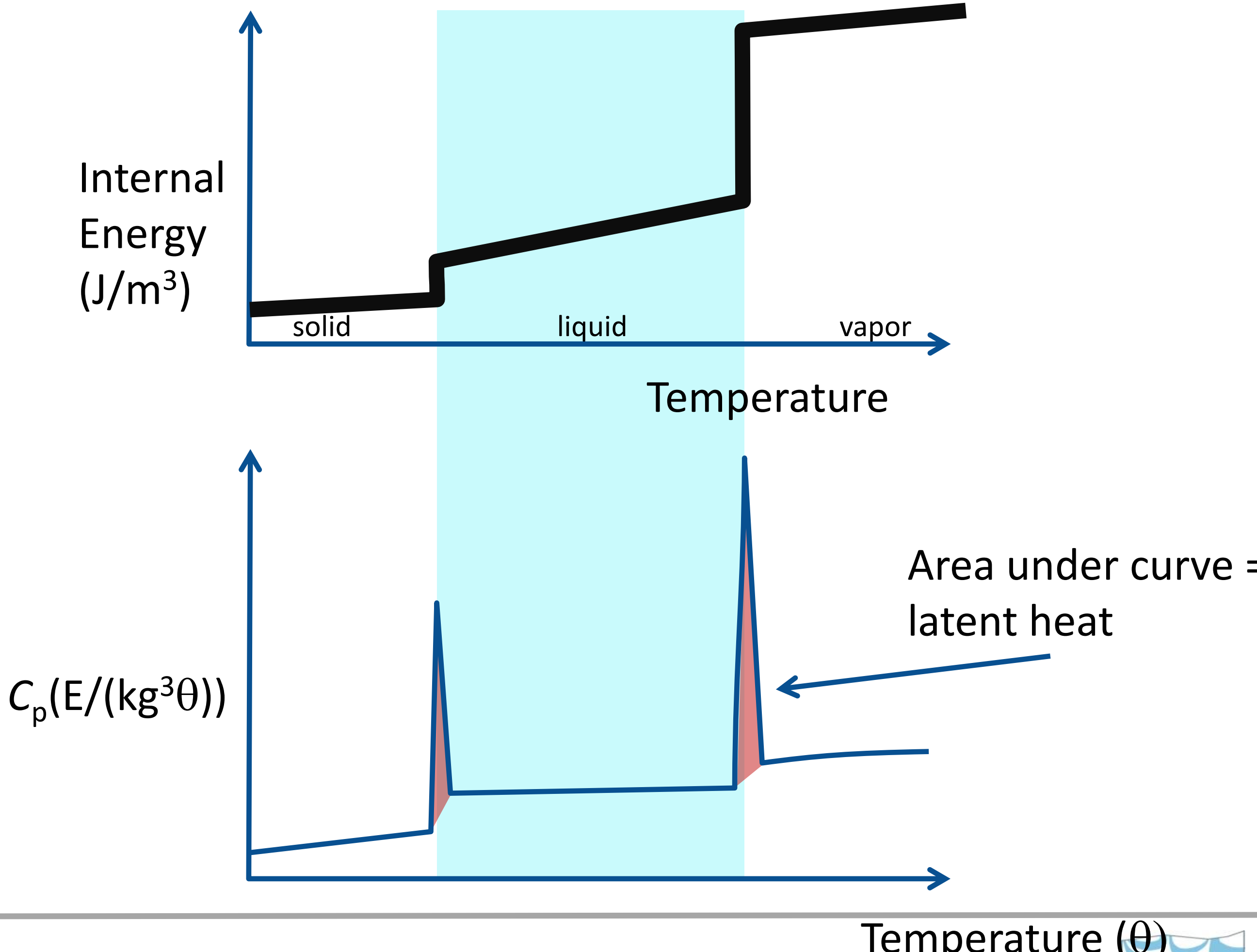
T: temperature

A: Max rate term



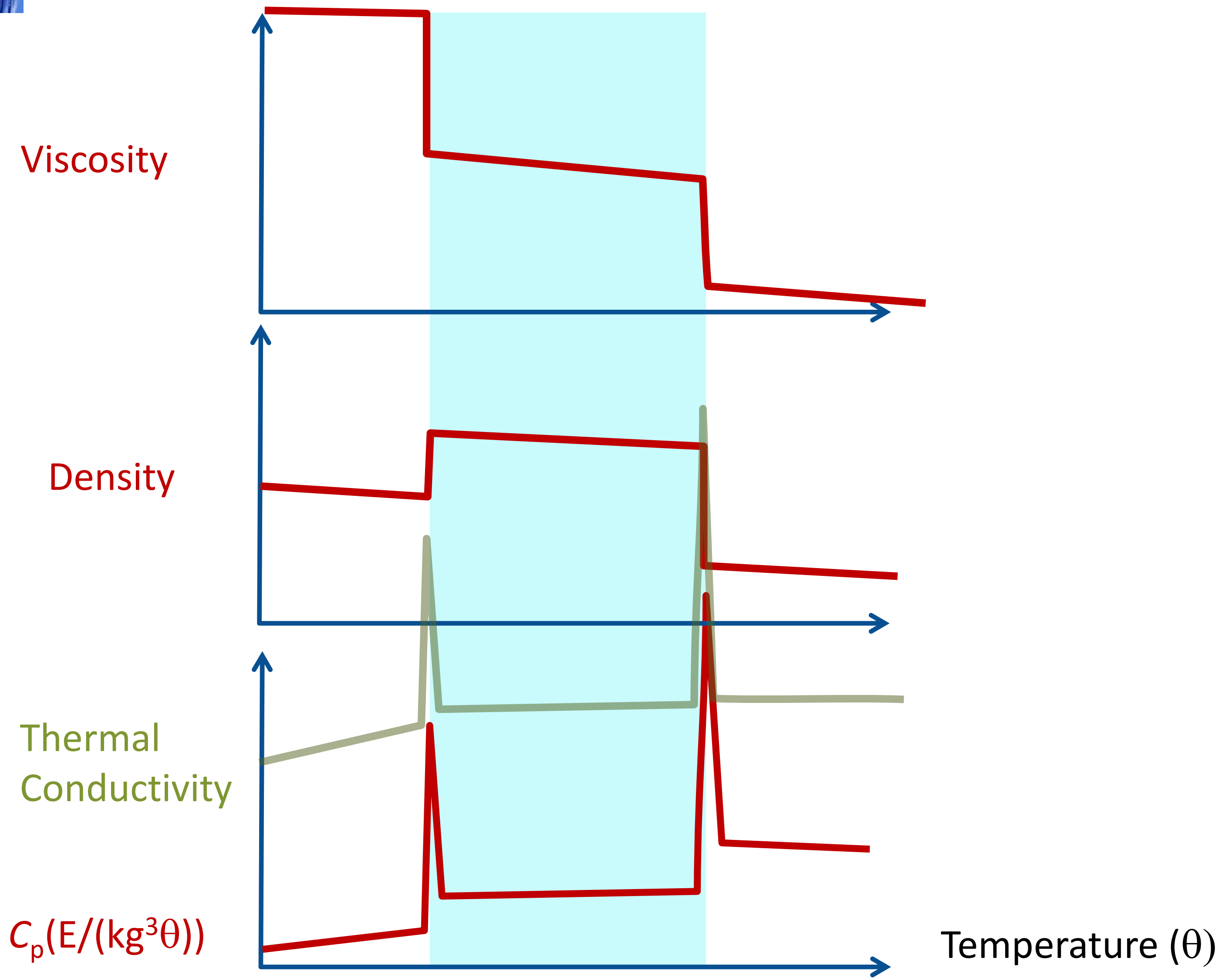
Including phase change

$$\nabla \cdot (-(K_h + D_h)\nabla T) + \nabla \cdot \mathbf{q} c_p \rho T + \frac{\partial c_p \rho n T}{\partial t} = \mathcal{S}_h$$



Clemson Hydro

Fluid properties



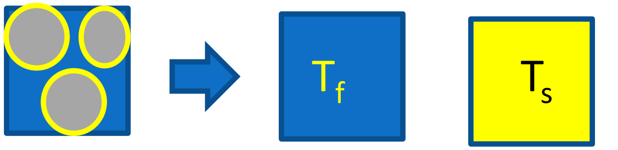
Thermal expansion

$$\alpha_V = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$$ Volumetric

$$\alpha_L = \frac{1}{L}\frac{dL}{dT}$$ Linear

Material	Linear coefficient, α, at 20 °C (10^{-6}/°C)	Volumetric coefficient, β, at 20 °C (10^{-6}/°C)
Aluminium	23.1	69
Aluminium nitride	5.3	4.2
Benzocyclobutene	42	126
Brass	19	57
Carbon steel	10.8	32.4
Concrete	12	36

Solid Phase in Porous Media



Heat loss by fluid

$$\frac{dE_{s,f}}{dt} = Q_{h,out} = q_h A = A\bar{h}(T_f - T_s)$$

$$\frac{dE_{s,f}}{V_c dt} = \frac{Q_{h,out}}{V_c} = q_h \frac{A}{V_c} = \frac{A}{V_c}\bar{h}(T_f - T_s)$$

Heat balance between fluid and solid

$$\frac{dE_{s,f}}{dt} = -\frac{dE_{s,s}}{dt}$$

$$\frac{d}{dt}\frac{E_{s,f}}{V_c} = -\frac{d}{dt}\frac{E_{s,s}}{V_c}$$

$$E_{s,f} = c_{pf}\rho_f n T_f \qquad \left[\frac{E_{s,f}}{M_f\theta}\frac{M_f}{L_f^3}\frac{L_f^3}{L_c^3}\frac{\theta}{\ }\right]$$

$$E_{s,s} = c_{ps}\rho_s (n-1) T_s \qquad \left[\frac{E_{s,s}}{M_s\theta}\frac{M_s}{L_s^3}\frac{L_s^3}{L_c^3}\frac{\theta}{\ }\right]$$

Material Properties

https://docs.google.com/viewer?url=http%3A%2F%2Fwww.bakker.org%2Fdartmouth06%2Fengs150%2F13-heat.pdf

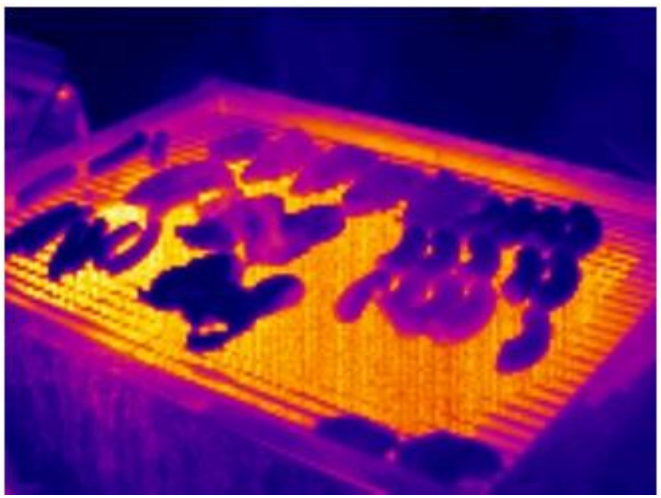
Heat Transfer
by Radiation and Convection

274 Meat refrigeration

Table 13.1 Mean thermal conductivities in chilling

	Mean thermal conductivity (W m^{-1} °C^{-1})	Variation with type
Lean meat	0.49	+0.05
(also kidney and liver)		
Fats		+0.02
Natural	0.21	
Rendered	0.15	
Bone		+0.02
compact bone	0.56	
spongy bone	0.26	
marrow	0.22	

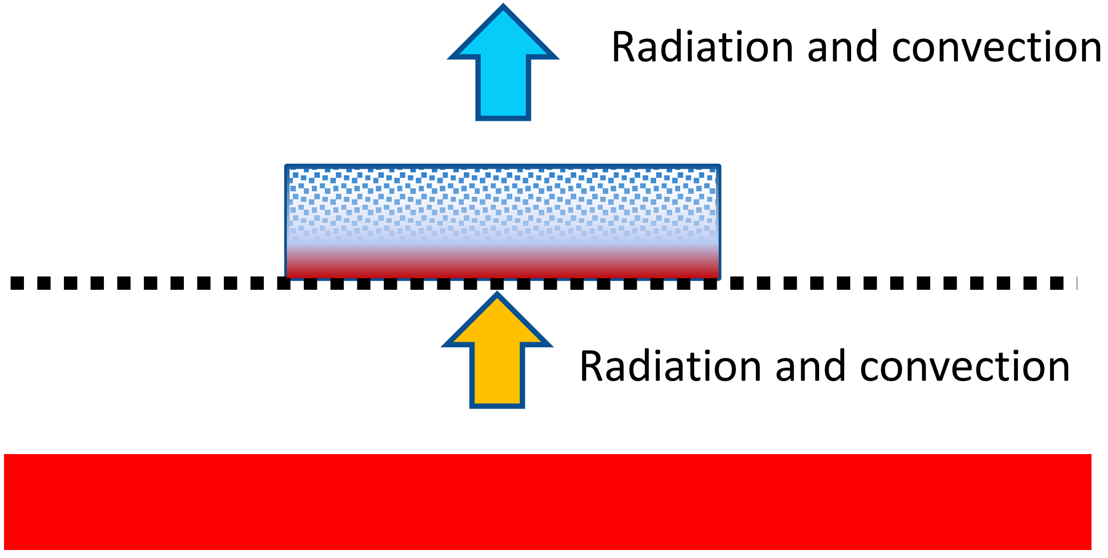
Source: Morley, 1972a.



Temperatures for beef, veal, lamb steaks and roasts

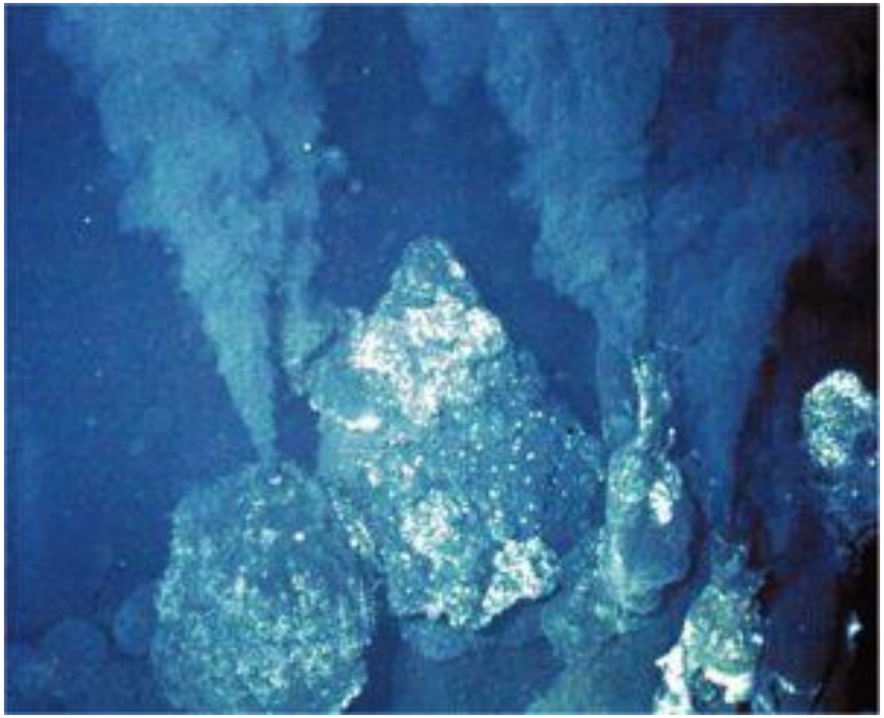
Term (*French*)	Description	Temperature range[1]		USDA recommended[2]
Extra-rare or Blue (*bleu*)	very red and cold	46–49 °C	115–120 °F	
Rare (*saignant*)	cold red center; soft	52–55 °C	125–130 °F	
Medium rare (*à point*)	warm red center; firmer	55–60 °C	130–140 °F	145 °F
Medium (*demi-anglais*)	pink and firm	60–65 °C	140–150 °F	160 °F
Medium well (*cuit*)	small amount of pink in the center	65–69 °C	150–155 °F	
Well done (*bien cuit*)	gray-brown throughout; firm	71–100 °C	160–212 °F	170 °F
Over cooked (*trop cuit*)	blacken throughout; hard	> 100 °C	> 212 °F	300 °F

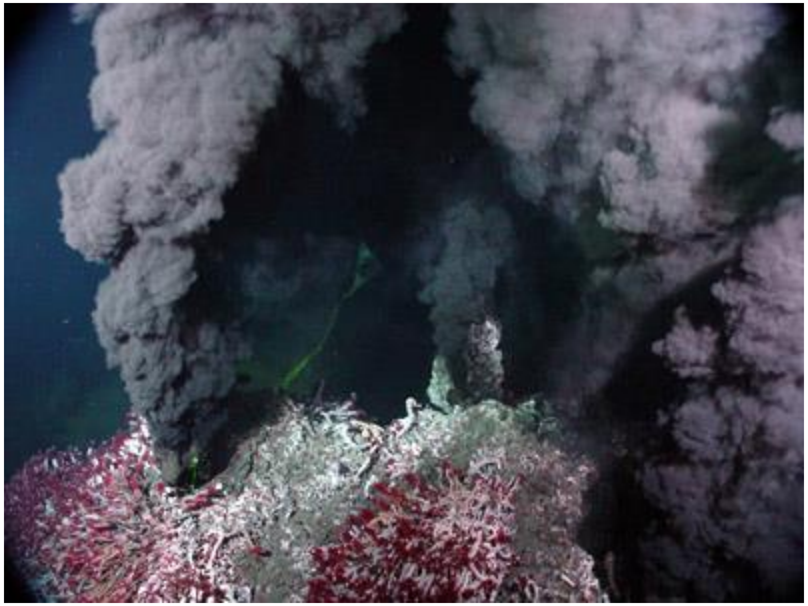
Conceptual Model



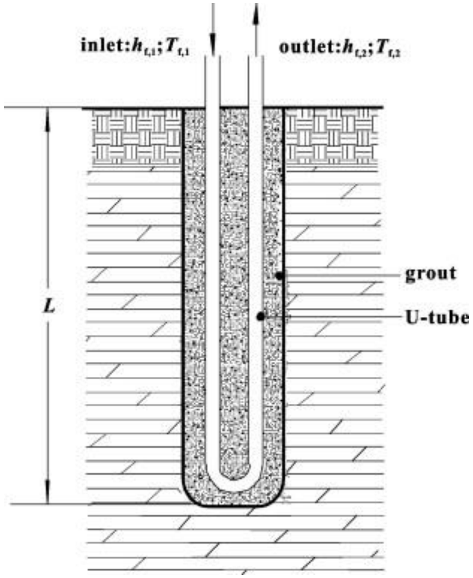
Radiogenic heat production ($\mu W/m^3$) of some rocks (from Fowler, *The Solid Earth*):

granite	2.5
average continental crust	1
tholeiitic basalt	0.08
average oceanic crust	0.5
peridotite	0.006
average undepleted mantle	0.02



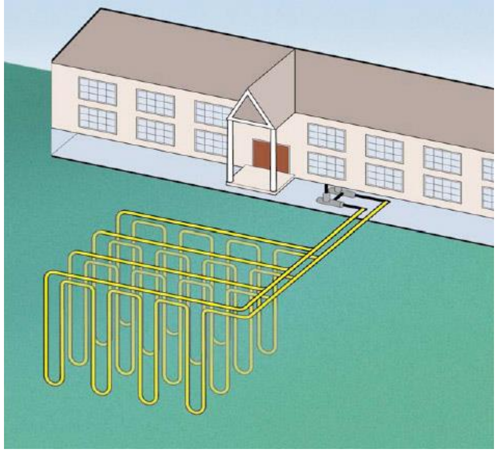


Installation of a U-tube in a borehole



U-tube heat exchanger

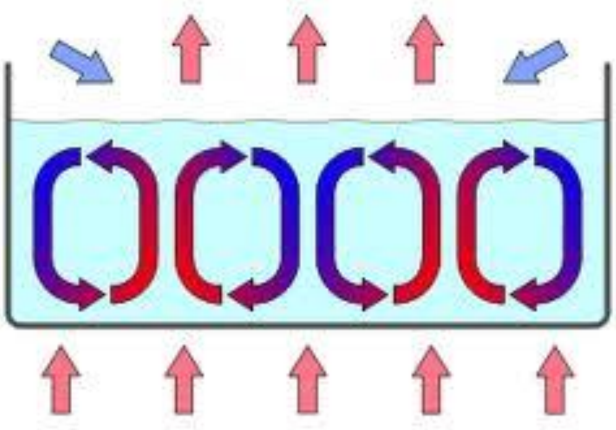
Two pipes from a U-tube heat exchanger in a borehole

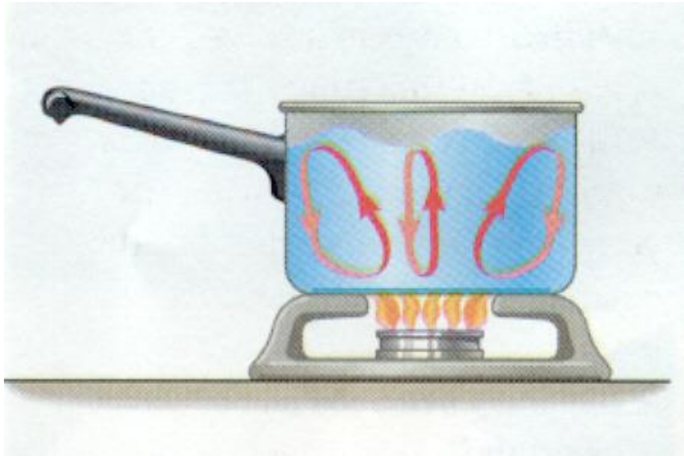


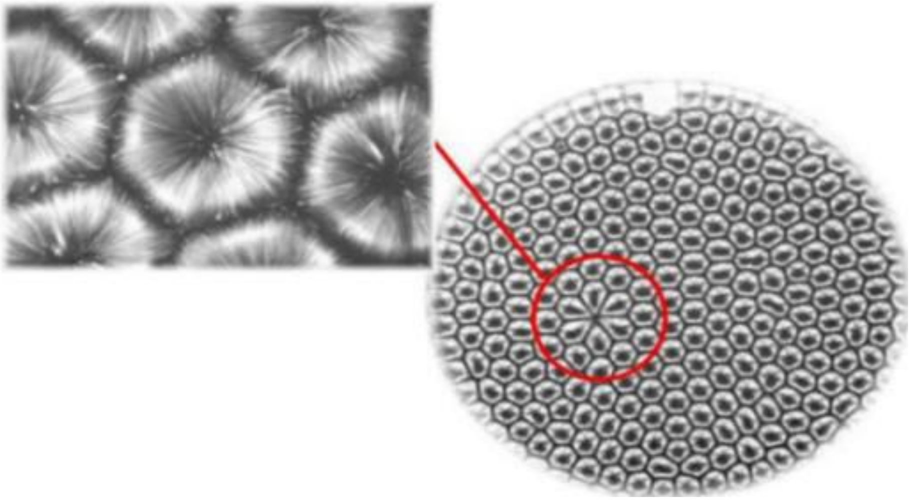
Array of U-tube heat exchangers

Free Convection

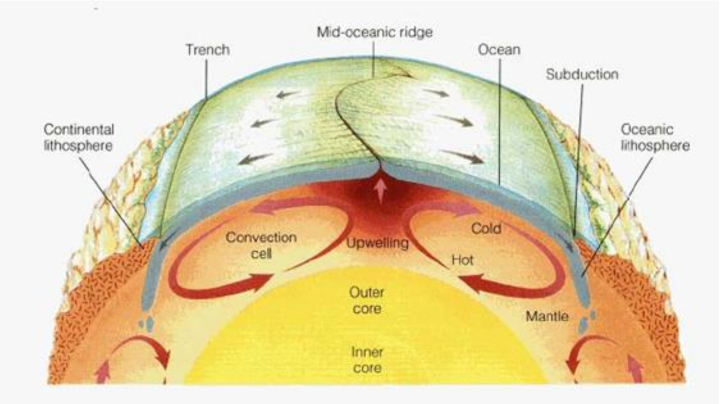
Rayleigh-Bernard Convection Cells

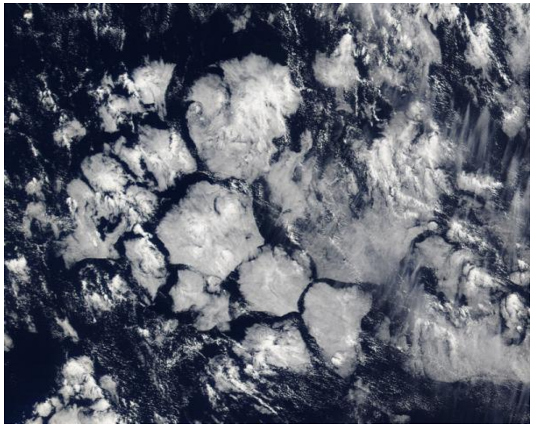






Convection in the Earth and Atm

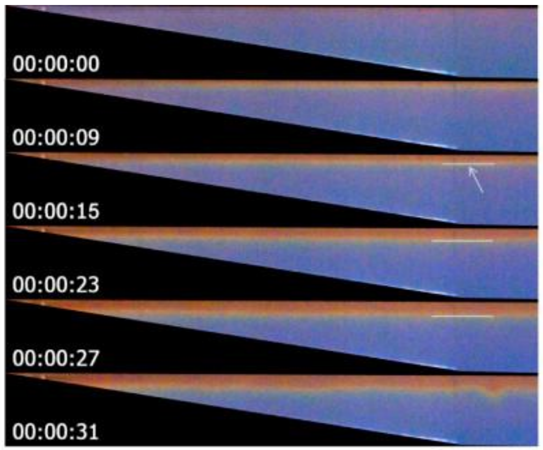




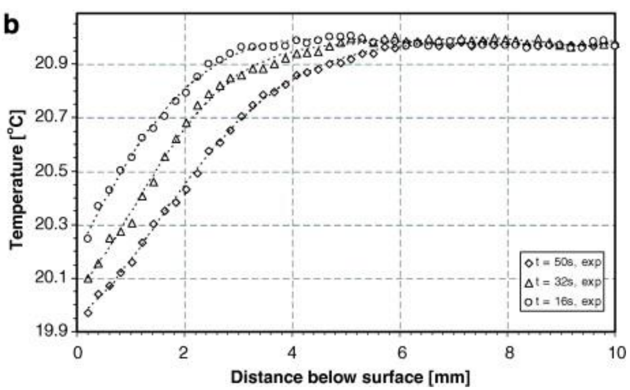
Experiment

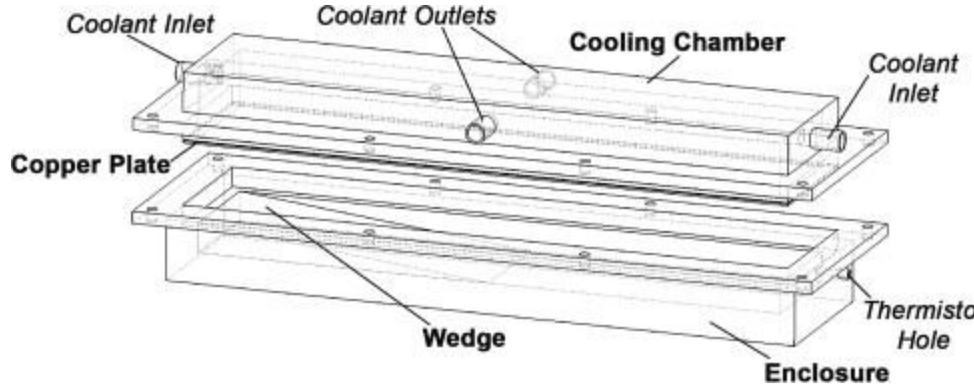
cooling at the top

a

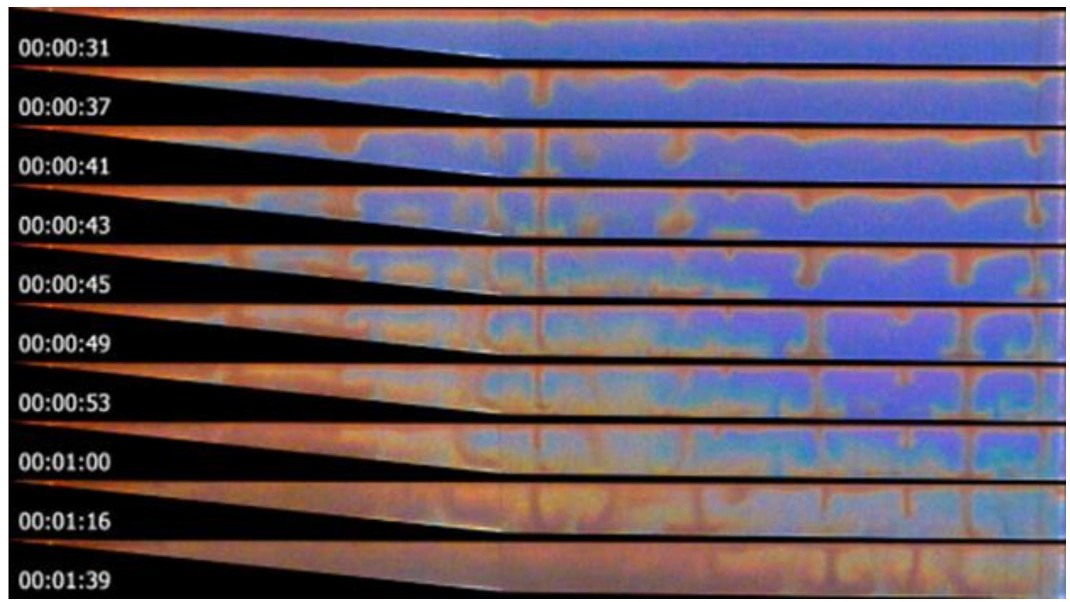


b





Simulated lake



http://www.sciencedirect.com/science/article/pii/S0894177707001409

Rayleigh number

$$Ra = \frac{\alpha g \Delta T d^3 c_p \rho^2}{\mu K_h}$$

α : thermal expansion

g: gravity

ΔT: temperature difference

d : layer thickness

c_p: heat capacity

ρ: fluid density

μ : dynamic viscosity

K_h : thermal conductivity

Onset of natural convection depends on Ra

For a layer with a free top surface: Ra_{crit}=1100

Coefficient of Thermal Expansion

$$\alpha_L = \frac{1}{L_o}\frac{\partial L}{\partial T}$$

$$\alpha_v = \frac{1}{V_o}\left(\frac{\partial V}{\partial T}\right)_P$$

$$\alpha_v = \frac{1}{\rho_o}\left(\frac{\partial \rho}{\partial T}\right)_P$$

Material	Linear coefficient, α, at 20 °C (10^{-6}/°C)	Volumetric coefficient, β, at 20 °C (10^{-6}/°C)
Aluminium	23.1	69
Aluminium nitride	5.3	4.2
Benzocyclobutene	42	126
Brass	19	57
Carbon steel	10.8	32.4
Concrete	12	36
Copper	17	51
Diamond	1	3
Ethanol	250	750[10]
Gallium(III) arsenide	5.8	17.4
Gasoline	317	950[9]
[illegible]	[illegible]	[illegible]
Titanium	8.6	
Tungsten	4.5	13.5
Water	69	207[13]
YbGaGe	≐0	≐0[20]
Zerodur	≈0.02	

Effect of varying temperature

on mixing in surface water